B. Sc. Physics Part-III
Mathematical Physics
Paper-I, Group-A

Complex Analysis Part-I

Complex number – $Z = x + iy$, $i = \sqrt{-1}$

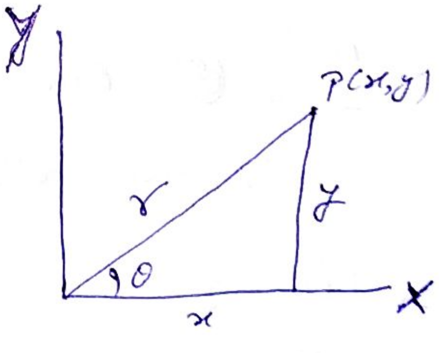
$x \rightarrow$ real part of the complex number Z
$y \rightarrow$ Imaginary part

* It is to be noted that imaginary part is not imaginary.

* x or y may be zero
If $x \rightarrow 0 \Rightarrow$ complex number is imaginary
If $y = 0 \Rightarrow$ Complex number is real.

Example $\rightarrow$ $3+4i$, $7+3i$, $9+10i$, $7i$, 3, etc

Complex plane –



$x + iy$

$x = r\cos\theta$
$y = r\sin\theta$

$x + iy = r\cos\theta + i r\sin\theta = r(\cos\theta + i\sin\theta) = re^{i\theta}$

Polar form representation.

Here we want to discuss the calculus of function Z, integration, differentiation, power series etc

As we have ~~been~~ seen that $z = x + iy$ is equivalent to pair of real numbers x, y, similarly we see that function of z is equivalent to a pair of real functions $u(x,y)$ and $v(x,y)$ of the real variables x and y, i.e. we can write function $f(z)$ as

$$\boxed{f(z) = f(x+iy) = u(x,y) + i\,v(x,y)}$$

Example: $f(z) = z^3$, find the real and imaginary parts $u(x,y)$ and $v(x,y)$.

Soln.

$$f(z) = z^3 = (x+iy)^3 = x^3 + (iy)^3 + 3ixy(x+iy)$$

or $f(z) = x^3 - iy^3 + 3ix^2y - 3xy^2$

or $f(z) = x^3 - 3xy^2 + i(3x^2y - y^3)$

Therefore $\quad u(x,y) = x^3 - 3xy^2; \quad v(x,y) = 3x^2y - y^3$

H.W. Find the real and imaginary parts $u(x,y)$ and $v(x,y)$ of the following functions

(i) z^2, (ii) $\bar{z}$, (iii) $\sin z$

(iv) e^z, (v) $\dfrac{z}{z^2+1}$, (vi) $\ln z$ (use $0 < \theta < 2\pi$)

(vii) e^{iz}, (viii) $\cosh z$, (ix) $\dfrac{1}{z}$

(x) $\sqrt{z}$